J Eng Math (2006) 56:201–202
DOI 10.1007/s10665-006-9078-6

PREFACE

Spectral interpolation in non-orthogonal domains: algorithms and applications

George Em Karniadakis · Jan S. Hesthaven

Received: 5 July 2006 / Accepted: 19 July 2006 /
Published online: 27 September 2006

Polynomial spectral methods have become increasingly important in diverse applications such as aeroacoustics, electromagnetics, ocean modeling, seismology, turbulent flows, non-Newtonian flows, non-linear optics, plasma dynamics, and uncertainty quantification. With more emphasis placed in the last 10 years on time-dependent rather than stationary problems, high-order methods, in general, and spectral-based methods, in particular, are potentially more effective than low-order discretizations. New developments in the last few years have addressed issues of complex geometry, finite regularity, fast solvers, multi-resolution, and large-scale simulations. The theme in the present special volume is spectral representations in non-orthogonal domains, specifically in triangles in two-dimensions and tetrahedra in three-dimensions.

Multi-dimensional spectral expansions in orthogonal domains (squares or cubes) are based on tensor-products of one-dimensional expansions; for example products of Chebyshev or Legendre polynomials are popular choices. Interpolation is typically based on grid points that are given by the roots of these polynomials (Gauss points) or by the roots of the polynomials' derivatives (Gauss–Lobatto points, if the end-points are included.) These grid points may also serve as the numerical quadrature points in spectral Galerkin formulations. However, in non-orthogonal domains, tensor-product expansion bases cannot be readily constructed, the best choice of grid points for interpolation or numerical integration is not so obvious. But good progress on both fronts has been made in the last 10 years, setting the foundation for a new generation of spectral and spectral-element methods that can handle truly complex geometries and can deal with error control very effectively. With regards to tensor products, new Jacobi-polynomial representations in conjunction with appropriate transformations have led to tensor-product bases in triangles and tetrahedra. With respect to the best set of grid points, several new algorithms have been proposed that provide excellent approximation properties in triangles and tetrahedra.

We have collected in this special volume eleven invited papers by leading researchers in spectral methods. They are divided in three groups. The first group (1–7) deals with fundamental approximation properties, the second group (8–9) addresses improvements in solution procedures, and the third group (10–11) presents applications.

G. Em Karniadakis (✉) · J. S. Hesthaven
Division of Applied Mathematics,
Brown University, Providence, RI, USA
e-mail: gk@dam.brown.edu

More specifically, the first paper by Boyd presents a review of methods as well as new ideas in constructing robust rootfinders for any smooth transcendental function. The second paper by Taylor and Wingate formulates a new Sturm–Liouville problem with generalized prolate spheroidal eigenfunctions on the triangle. The third paper by Wingate and Taylor introduces a new measure of resolution of spectral multi-dimensional polynomials and applies it to the class of orthogonal polynomials presented in the second paper. The fourth paper by Warburton develops a simple explicit procedure in constructing interpolation grid points on the triangle and tetrahedron that compares well with other approaches. The fifth paper by Blyth et al. presents a comparison of interpolation grids on the triangle and tetrahedron, with the grids generated on a one-dimensional master grid comprised of the roots of spectral polynomials. The sixth paper by Kirby and Sherwin addresses aliasing errors in quadratic non-linearities and compares rotationally non-symmetric and symmetric collocation grids. The seventh paper by Grinberg and Karniadakis investigates the use of different types of grid points as quadrature grids for the construction of operators involved in the Galerkin formulation of the Navier–Stokes equations.

In the second group, the eighth paper by Giraldo and Taylor discusses the use of cubature points for interpolations through careful filtering of the basis. This approach results in diagonal mass-matrices and explicit time-integration that performs well, as shown through a few examples. The ninth paper by Pasquetti et al., develops new preconditioners for elliptic problems arising in spectral element discretizations where Fekete grid points are employed.

In the third group, the tenth paper by Heinrichs discusses the formulation and application of a Least-Squares stabilized method for adaptive solution of the incompressible Navier–Stokes equations on unstructured grids. Finally, the eleventh paper by Engsig-Karup et al. develops a nodal discontinuous Galerkin method for solving a class of high-order non-linear Boussinesq models for free-surface flows. The scheme is carefully validated by comparison against simple solutions as well as experiments.

We hope that the readers find this special volume useful. We also recommend the upcoming International Conference on Spectral and High-Order Methods 2007 (ICOSAHOM'07), taking place in Beijing, PRC, during June 18–22, 2007, as an opportunity and meeting place for researchers in the topic of this special volume. Interested readers can search for other topics in spectral and high-order methods in all six previous ICOSAHOM conferences.

Finally, we would like to thank all contributors as well as the Journal Editor Professor C. Pozrikidis who gave us the opportunity to put together this special volume.

George Em Karniadakis and Jan S. Hesthaven
Guest Editors